## Exercise 3D: Solutions

**1 a**

$$\frac{5x+1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$
$$= \frac{A(x+2)+B(x-1)}{(x-1)(x+2)}$$
$$= \frac{Ax+Bx+2A-B}{(x-1)(x+2)}$$
$$A+B=5 \quad (1)$$
$$2A-B=1 \quad (2)$$
$(1)+(2)$:
$$3A=6$$
$$A=2$$
$$2+B=5$$
$$B=3$$
$$\therefore \frac{5x+1}{(x-1)(x+2)} = \frac{2}{x-1} + \frac{3}{x+2}$$

**b**

$$\frac{-1}{(x+1)(2x+1)} = \frac{A}{x+1} + \frac{B}{2x+1}$$
$$= \frac{A(2x+1)+B(x+1)}{(x+1)(2x+1)}$$
$$= \frac{2Ax+Bx+A+B}{(x+1)(2x+1)}$$
$$2A+B=0 \quad (1)$$
$$A+B=-1 \quad (2)$$
$(1)-(2)$:
$$A=1$$
$$1+B=-1$$
$$B=-2$$
$$\therefore \frac{-1}{(x+1)(2x+1)} = \frac{1}{x+1} - \frac{2}{2x+1}$$

**c**

$$\frac{3x-2}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2}$$
$$= \frac{A(x-2)+B(x+2)}{(x+2)(x-2)}$$
$$= \frac{Ax+Bx-2A+2B}{(x+2)(x-2)}$$
$$A+B=3$$
$$2A+2B=6 \quad (1)$$
$$-2A+2B=-2 \quad (2)$$
$(1)+(2)$:
$$4B=4$$
$$B=1$$
$$A+1=3$$
$$A=2$$
$$\therefore \frac{3x-2}{(x+2)(x-2)} = \frac{2}{x+2} + \frac{1}{x-2}$$

**d**

$$\begin{aligned}\frac{4x+7}{(x+3)(x-2)} &= \frac{A}{x+3}+\frac{B}{x-2}\\ &= \frac{A(x-2)+B(x+3)}{(x+3)(x-2)}\\ &= \frac{Ax+Bx-2A+3B}{(x+3)(x-2)}\end{aligned}$$

$$\begin{aligned}A+B &= 4\\ 2A+2B &= 8 \quad (1)\\ -2A+3B &= 7 \quad (2)\end{aligned}$$

$(1)+(2)$:

$$\begin{aligned}5B &= 15\\ B &= 3\\ A+3 &= 4\\ A &= 1\end{aligned}$$

$$\therefore \frac{4x+7}{(x+3)(x-2)} = \frac{1}{x+3}+\frac{3}{x-2}$$

**e**

$$\begin{aligned}\frac{7-x}{(x-4)(x+1)} &= \frac{A}{x-4}+\frac{B}{x+1}\\ &= \frac{A(x+1)+B(x-4)}{(x-4)(x+1)}\\ &= \frac{Ax+Bx+A-4B}{(x-4)(x+1)}\end{aligned}$$

$$\begin{aligned}A+B &= -1 \quad (1)\\ A-4B &= 7 \quad (2)\end{aligned}$$

$(1)-(2)$:

$$\begin{aligned}5B &= -8\\ B &= -\frac{8}{5}\\ A-\frac{8}{5} &= -1\\ A &= \frac{3}{5}\end{aligned}$$

$$\therefore \frac{7-x}{(x-4)(x+1)} = \frac{3}{5(x-4)}-\frac{8}{5(x+1)}$$

**2 a**

$$\begin{aligned}\frac{2x+3}{(x-3)^2} &= \frac{A}{x-3}+\frac{B}{(x-3)^2}\\ &= \frac{A(x-3)+B}{(x-3)^2}\\ &= \frac{Ax-3A+B}{(x-3)^2}\end{aligned}$$

$$\begin{aligned}A &= 2\\ -3A+B &= 3\\ -6+B &= 3\\ B &= 9\end{aligned}$$

$$\therefore \frac{2x+3}{(x-3)^2} = \frac{2}{x-3}+\frac{9}{(x-3)^2}$$

**b**

$$\begin{aligned}\frac{9}{(1+2x)(1-x)^2} &= \frac{A}{1+2x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}\\ &= \frac{A(1-x)^2+B(1+2x)(1-x)+C(1+2x)}{(1+2x)(1-x)^2}\\ &= \frac{A-2Ax+Ax^2+B+Bx-2Bx^2+C+2Cx}{(1+2x)(1-x)^2}\end{aligned}$$

$$\begin{aligned}A-2B &= 0 && (1)\\ -2A+B+2C &= 0 && (2)\\ A+B+C &= 9 && (3)\\ 2A+2B+2C &= 18 && (4)\\ (4)-(2)&:\\ 4A+B &= 18\\ (1)\times(4)&: 4A-8B = 0\\ 9B &= 18\\ B &= 2\\ 4A+2 &= 18\\ A &= 4\\ 4+2+C &= 9\\ C &= 3\end{aligned}$$

$$\therefore \quad \frac{9}{(1+2x)(1-x)^2} = \frac{4}{1+2x}+\frac{2}{1-x}+\frac{3}{(1-x)^2}$$

**c**

$$\begin{aligned}\frac{2x-2}{(x+1)(x-2)^2} &= \frac{A}{(x+1)}+\frac{B}{(x-2)}+\frac{C}{(x-2)^2}\\ &= \frac{A(x-2)^2+B(x+1)(x-2)+C(x+1)}{(x+1)(x-2)^2}\\ &= \frac{Ax^2-4Ax+4A+Bx^2-Bx-2B+Cx+C}{(x+1)(x-2)^2}\end{aligned}$$

$$\begin{aligned}A+B &= 0 && (1)\\ -4A-B+C &= 2 && (2)\\ 4A-2B+C &= -2 && (3)\\ (3)-(2)&: 8A-B = -4 && (4)\\ (4)+(1)&: 9A = -4\\ A &= -\frac{4}{9}\\ A+B &= 0\\ B &= \frac{4}{9}\\ 4A-2B+C &= -2\\ -\frac{16}{9}-\frac{8}{9}+C &= -2\\ C &= -2+\frac{24}{9} = \frac{2}{3}\end{aligned}$$

$$\therefore \quad \frac{2x-2}{(x+1)(x-2)^2} = -\frac{4}{9(x+1)}+\frac{4}{9(x-2)}+\frac{2}{3(x-2)^2}$$

**3 a**

$$\frac{3x+1}{(x+1)(x^2+x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+x+1}$$
$$= \frac{A(x^2+x+1)+(Bx+C)(x+1)}{(x+1)(x^2+x+1)}$$
$$= \frac{Ax^2+Ax+A+Bx^2+Bx+Cx+C}{(x+1)(x^2+x+1)}$$

$$A+B=0 \quad (1)$$
$$A+B+C=3 \quad (2)$$
$$A+C=1 \quad (3)$$
$$(2)-(1): C=3$$
$$A+3=1$$
$$A=-2$$
$$A+B+C=3$$
$$-2+B+3=3$$
$$B=2$$

$$\therefore \frac{3x+1}{(x+1)(x^2+x+1)} = -\frac{2}{x+1} + \frac{2x+3}{x^2+x+1}$$

**b**

$$\frac{3x^2+2x+5}{(x^2+2)(x+1)} = \frac{Ax+B}{x^2+2} + \frac{C}{x+1}$$
$$= \frac{(Ax+B)(x+1)+C(x^2+2)}{(x^2+2)(x+1)}$$
$$= \frac{Ax^2+Ax+Bx+B+Cx^2+2C}{(x^2+2)(x+1)}$$

$$A+C=3 \quad (1)$$
$$A+B=2 \quad (2)$$
$$B+2C=5 \quad (3)$$
$$(1)-(2):$$
$$C-B=1 \quad (4)$$
$$(3)+(4):$$
$$3C=6$$
$$C=2$$
$$A+2=3$$
$$A=1$$
$$1+B=2$$
$$B=1$$

$$\therefore \frac{3x^2+2x+5}{(x^2+2)(x+1)} = \frac{x+1}{x^2+2} + \frac{2}{x+1}$$

**c** Factorise the denominator:

$$2x^3+6x^2+2x+6 = 2x^2(x+3)+2(x+3)$$
$$= 2(x^2+1)(x+3)$$

The 2 factor can be put with either fraction.

$$\frac{x^2+2x-13}{2(x^2+1)(x+3)} = \frac{Ax+B}{x^2+1} + \frac{C}{2(x+3)}$$
$$= \frac{2(Ax+B)(x+3)+C(x^2+1)}{2(x^2+1)(x+3)}$$
$$= \frac{2Ax^2+6Ax+2Bx+6B+Cx^2+C}{2(x^2+1)(x+3)}$$

$$2A+C=1 \quad (1)$$
$$6A+2B=2$$
$$9A+3B=3 \quad (2)$$
$$6B+C=-13 \quad (3)$$

(1) − (3):

$$2A-6B=14$$
$$A-3B=7 \quad (4)$$

(2) + (4):

$$10A=10$$
$$A=1$$
$$2+C=1$$
$$C=-1$$
$$3A+B=1$$
$$A+B=1$$
$$B=-2$$

$$\therefore \quad \frac{x^2+2x-13}{2(x^2+1)(x+3)} = \frac{x-2}{x^2+1} - \frac{1}{2(x+3)}$$

**4** $(x-1)(x-2) = x^2-3x+2$

First divide:

$$3x^2-4x-2 = 3(x^2-3x+2)+5x-8$$
$$\frac{3x^2-4x-2}{(x-1)(x-2)} = \frac{5x-8}{(x-1)(x-2)} + 3$$
$$\frac{5x-8}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$
$$= \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$
$$= \frac{Ax+Bx-2A-B}{(x-1)(x-2)}$$

$$A+B=5 \quad (1)$$
$$-2A-B=-8 \quad (2)$$

(1) + (2):

$$-A=-3$$
$$A=3$$
$$3+B=5$$
$$B=2$$

$$\therefore \quad \frac{5x-8}{(x-1)(x-2)} = \frac{3}{x-1} + \frac{2}{x-2}$$

Use the previous working:

$$\frac{3x^2-4x-2}{(x-1)(x-2)} = 3 + \frac{3}{x-1} + \frac{2}{x-2}$$

**5**

$$\begin{aligned}\frac{2x+10}{(x+1)(x-1)^2} &= \frac{A}{x+1}+\frac{C}{(x-1)^2}\\ &= \frac{A(x-1)^2+C(x+1)}{(x+1)(x-1)^2}\\ &= \frac{Ax^2-2Ax+A+Cx+C}{(x+1)(x-1)^2}\end{aligned}$$

$$\begin{aligned}A &= 0\\ -2A+C &= 2\\ C &= 2\\ A+C &= 10\\ 0+2 &\neq 10\end{aligned}$$

It is impossible to find $A$ and $C$ to satisfy this equation.

**6 a**

$$\begin{aligned}\frac{1}{(x-1)(x+1)} &= \frac{A}{x-1}+\frac{B}{x+1}\\ &= \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}\\ &= \frac{Ax+Bx+A-B}{(x-1)(x+1)}\end{aligned}$$

$$A+B=0 \quad (1)$$

$$A-B=1 \quad (2)$$

$(1)+(2)$:

$$\begin{aligned}2A &= 1\\ A &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{1}{2}+B &= 0\\ B &= -\frac{1}{2}\end{aligned}$$

$$\therefore \quad \frac{1}{(x-1)(x+1)} = \frac{1}{2(x-1)}-\frac{1}{2(x+1)}$$

**b**

$$\begin{aligned}\frac{x}{(x-2)(x+3)} &= \frac{A}{x-2}+\frac{B}{x+3}\\ &= \frac{A(x+3)+B(x-2)}{(x-2)(x+3)}\\ &= \frac{Ax+Bx+3A-2B}{(x-2)(x+3)}\end{aligned}$$

$$A+B=1$$

$$2A+2B=2 \quad (1)$$

$$3A-2B=0 \quad (2)$$

$(1)+(2)$:

$$\begin{aligned}5A &= 2\\ A &= \frac{2}{5}\end{aligned}$$

$$\begin{aligned}\frac{2}{5}+B &= 1\\ B &= \frac{3}{5}\end{aligned}$$

$$\therefore \quad \frac{x}{(x-2)(x+3)} = \frac{2}{5(x-2)}+\frac{3}{5(x+3)}$$

**c**

$$\begin{aligned}\frac{3x+1}{(x-2)(x+5)} &= \frac{A}{x-2}+\frac{B}{x+5}\\ &= \frac{A(x+5)+B(x-2)}{(x-2)(x+5)}\\ &= \frac{Ax+Bx+5A-2B}{(x-2)(x+5)}\end{aligned}$$

$$\begin{aligned}A+B &= 3\\ 2A+2B &= 6 \quad (1)\\ 5A-2B &= 1 \quad (2)\\ (1)+(2)&:\\ 7A &= 7\\ A &= 1\\ 1+B &= 3\\ B &= 2\end{aligned}$$

$$\therefore \quad \frac{3x+1}{(x-2)(x+5)} = \frac{1}{x-2}+\frac{2}{x+5}$$

**d**

$$\begin{aligned}\frac{1}{(2x-1)(x+2)} &= \frac{A}{2x-1}+\frac{B}{x+2}\\ &= \frac{A(x+2)+B(2x-1)}{(2x-1)(x+2)}\\ &= \frac{Ax+2Bx+2A-B}{(2x-1)(x+2)}\end{aligned}$$

$$\begin{aligned}A+2B &= 0\\ 2A+4B &= 0 \quad (1)\\ 2A-B &= 1 \quad (2)\\ (1)+(2)&:\\ 5B &= -1\\ B &= -\frac{1}{5}\\ A+2B &= 0\\ A &= \frac{2}{5}\end{aligned}$$

$$\therefore \quad \frac{1}{(2x-1)(x+2)} = \frac{2}{5(2x-1)}-\frac{1}{5(x+2)}$$

**e**

$$\begin{aligned}\frac{3x+5}{(3x-2)(2x+1)} &= \frac{A}{3x-2}+\frac{B}{2x+1}\\ &= \frac{A(2x+1)+B(3x-2)}{(3x-2)(2x+1)}\\ &= \frac{2Ax+3Bx+A-2B}{(3x-2)(2x+1)}\end{aligned}$$

$$\begin{aligned}2A+3B &= 3 \quad (1)\\ A-2B &= 5\\ 2A-4B &= 10 \quad (2)\\ (1)-(2)&:\\ 7B &= -7\\ B &= -1\\ A-2\times -1 &= 5\\ A &= 3\end{aligned}$$

$$\therefore \quad \frac{3x+5}{(3x-2)(2x+1)} = \frac{3}{3x-2}-\frac{1}{2x+1}$$

**f**

$$\begin{aligned}\frac{2}{x(x-1)} &= \frac{A}{x} + \frac{B}{x-1} \\ &= \frac{A(x-1)+Bx}{x(x-1)} \\ &= \frac{Ax+Bx-A}{x(x-1)}\end{aligned}$$

$$\begin{aligned}A+B &= 0 \\ -A &= 2 \\ A &= -2 \\ -2+B &= 0 \\ B &= 2\end{aligned}$$

$$\therefore \quad \frac{2}{x(x-1)} = \frac{2}{x-1} - \frac{2}{x}$$

**g**

$$\begin{aligned}\frac{3x+1}{x(x^2+1)} &= \frac{A}{x} + \frac{Bx+C}{x^2+1} \\ &= \frac{A(x^2+1)+x(Bx+C)}{x(x^2+1)} \\ &= \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)}\end{aligned}$$

$$\begin{aligned}A+B &= 0 \\ C &= 3 \\ A &= 1 \\ 1+B &= 0 \\ B &= -1\end{aligned}$$

$$\therefore \quad \frac{3x+1}{x(x^2+1)} = \frac{1}{x} + \frac{3-x}{x^2+1}$$

**h**

$$\begin{aligned}\frac{3x^2+8}{x(x^2+4)} &= \frac{A}{x} + \frac{Bx+C}{x^2+4} \\ &= \frac{A(x^2+4)+x(Bx+C)}{x(x^2+4)} \\ &= \frac{Ax^2+4A+Bx^2+Cx}{x(x^2+4)}\end{aligned}$$

$$\begin{aligned}A+B &= 3 \\ C &= 0 \\ 4A &= 8 \\ A &= 2 \\ 2+B &= 3 \\ B &= 1\end{aligned}$$

$$\therefore \quad \frac{3x^2+8}{x(x^2+4)} = \frac{2}{x} + \frac{x}{x^2+4}$$

**i**

$$\begin{aligned}\frac{1}{x(x-4)} &= \frac{A}{x} + \frac{B}{x-4} \\ &= \frac{A(x-4)+Bx}{x(x-4)} \\ &= \frac{Ax+Bx-4A}{x(x-4)}\end{aligned}$$

$$\begin{aligned}A+B &= 0 \\ -4A &= 1 \\ A &= -\frac{1}{4}\end{aligned}$$

$$-\frac{1}{4}+B=0$$

$$B=\frac{1}{4}$$

$$\therefore \quad \frac{1}{x(x-4)}=\frac{1}{4(x-4)}-\frac{1}{4x}$$

**j**

$$\frac{x+3}{x(x-4)}=\frac{A}{x}+\frac{B}{x-4}$$

$$=\frac{A(x-4)+Bx}{x(x-4)}$$

$$=\frac{Ax+Bx-4A}{x(x-4)}$$

$$A+B=1$$

$$-4A=3$$

$$A=-\frac{3}{4}$$

$$-\frac{3}{4}+B=1$$

$$B=\frac{7}{4}$$

$$\therefore \quad \frac{x+3}{x(x-4)}=\frac{7}{4(x-4)}-\frac{3}{4x}$$

**k** First divide $x^2-x^2-1$ by $x^2-x$.
You might observe a pattern in the question.

$$\frac{x^3-x^2-1}{x^2-x}=\frac{x(x^2-x)-1}{x^2-x}=x-\frac{1}{x^2-x}$$

Express $-\dfrac{1}{x^2-x}$ in partial fractions.

$$-\frac{1}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}$$

$$=\frac{A(x-1)+Bx}{x(x-1)}$$

$$=\frac{Ax+Bx-A}{x(x-1)}$$

$$A+B=0$$

$$-A=-1$$

$$A=1$$

$$1+B=0$$

$$B=-1$$

$$\therefore \quad \frac{-1}{x(x-1)}=\frac{1}{x}-\frac{1}{x-1}$$

$$\frac{x^3-x^2-1}{x^2-x}=x+\frac{1}{x}-\frac{1}{x-1}$$

**l** First divide $(x^2-x^2-6)$ by $(-x^2+2x)$.

$$\begin{array}{r|l} & -x-1 \\ \hline -x^2+2x & x^3-x^2-6 \\ & \underline{x^3-2x^2} \\ & x^2-6 \\ & \underline{x^2-2x} \\ & 2x-6 \end{array}$$

$$\therefore \quad (x^3 - x^2 - 6) \div (-x^2 + 2x) = -x - 1 + \frac{2x - 6}{x(2 - x)}$$

Separate $\dfrac{2x - 6}{x(2 - x)}$ into partial fractions.

$$\begin{aligned}
\frac{2x - 6}{x(2 - x)} &= \frac{A}{x} + \frac{B}{2 - x} \\
&= \frac{A(2 - x) + Bx}{x(2 - x)} \\
&= \frac{-Ax + Bx + 2A}{x(2 - x)} \\
-A + B &= 2 \\
2A &= -6 \\
A &= -3 \\
3 + B &= 2 \\
B &= -1
\end{aligned}$$

$$\therefore \quad \frac{2x - 6}{x(2 - x)} = -\frac{3}{x} - \frac{1}{2 - x}$$

$$\frac{x^3 - x^2 - 6}{2x - x^2} = -x - 1 - \frac{3}{x} - \frac{1}{2 - x}$$

**m**

$$\begin{aligned}
\frac{x^2 - x}{(x + 1)(x^2 + 2)} &= \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 2} \\
&= \frac{A(x^2 + 2) + (Bx + C)(x + 1)}{(x + 1)(x^2 + 2)} \\
&= \frac{Ax^2 + 2A + Bx^2 + Bx + Cx + C}{(x + 1)(x^2 + 2)} \\
A + B &= 1 \qquad (1) \\
B + C &= -1 \qquad (2) \\
2A + C &= 0 \qquad (3) \\
(1) - (2): A - C &= 2 \qquad (4) \\
(3) + (4): 3A &= 2 \\
A &= \frac{2}{3} \\
\frac{2}{3} + B &= 1 \\
B &= \frac{1}{3} \\
\frac{1}{3} + C &= -1 \\
C &= -\frac{4}{3}
\end{aligned}$$

$$\therefore \quad \frac{x^2 - x}{(x + 1)(x^2 + 2)} = \frac{2}{3(x + 1)} + \frac{x - 4}{3(x^2 + 2)}$$

**n** $x^3 - 3x - 2$ can be factorised into $(x-2)(x+1)^2$.

$$\begin{aligned}\frac{x^2+2}{(x-2)(x+1)^2} &= \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \\ &= \frac{A(x+1)^2 + B(x+1)(x-2) + C(x-2)}{(x-2)(x+1)^2} \\ &= \frac{Ax^2 + 2Ax + A + Bx^2 - Bx - 2B + Cx - 2C}{(x-2)(x+1)^2}\end{aligned}$$

$$\begin{aligned}A + B &= 1 \quad (1) \\ 2A - B + C &= 0 \\ 4A - 2B + 2C &= 0 \quad (2) \\ A - 2B - 2C &= 2 \quad (3) \\ (2) + (3)&: \\ 5A - 4B &= 2 \quad (4) \\ (4) - 4 \times (1)&: \\ 9A &= 6 \\ A &= \frac{2}{3} \\ A + B &= 1 \\ B &= \frac{1}{3} \\ \frac{4}{3} - \frac{1}{3} + C &= 0 \\ C &= -1\end{aligned}$$

$$\therefore \quad \frac{x^2+2}{(x-2)(x+1)^2} = \frac{2}{3(x-2)} + \frac{1}{3(x+1)} - \frac{1}{(x+1)^2}$$

**o**

$$\begin{aligned}\frac{2x^2+x+8}{x(x^2+4)} &= \frac{A}{x} + \frac{Bx+C}{x^2+4} \\ &= \frac{A(x^2+4) + x(Bx+C)}{x(x^2+4)} \\ &= \frac{Ax^2 + 4A + Bx^2 + Cx}{x(x^2+4)}\end{aligned}$$

$$\begin{aligned}A + B &= 2 \\ C &= 1 \\ 4A &= 8 \\ A &= 2 \\ 2 + B &= 2 \\ B &= 0\end{aligned}$$

$$\therefore \quad \frac{2x^2+x+8}{x(x^2+4)} = \frac{2}{x} + \frac{1}{x^2+4}$$

**p** $2x^2 + 7x + 6 = (2x+3)(x+2)$

$$\begin{aligned}\frac{1-2x}{(2x+3)(x+2)} &= \frac{A}{2x+3} + \frac{B}{x+2} \\ &= \frac{A(x+2) + B(2x+3)}{(2x+3)(x+2)} \\ &= \frac{Ax + 2Bx + 2A + 3B}{(2x+3)(x+2)}\end{aligned}$$

$$\begin{aligned}A + 2B &= -2 \\ 2A + 4B &= -4 \quad (1) \\ 2A + 3B &= 1 \quad (2)\end{aligned}$$

$$
\begin{aligned}
\text{①} - \text{②}&: \\
B &= -5 \\
A + 2 \times -5 &= -2 \\
A &= 8 \\
\therefore \quad \frac{1-2x}{(2x+3)(x+2)} &= \frac{8}{2x+3} - \frac{5}{x+2}
\end{aligned}
$$

**q**

$$
\begin{aligned}
\frac{3x^2-6x+2}{(x-1)^2(x+2)} &= \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \\
&= \frac{A(x-1)^2 + B(x+2)(x-1) + C(x+2)}{(x-1)^2(x+2)} \\
&= \frac{Ax^2 - 2Ax + A + Bx^2 + Bx - 2B + Cx + 2C}{(x-1)^2(x+2)}
\end{aligned}
$$

$$
\begin{aligned}
A + B &= 3 \\
4A + 4B &= 12 \qquad \text{①} \\
-2A + B + C &= -6 \qquad \text{②} \\
A - 2B + 2C &= 2 \qquad \text{③} \\
\text{③} - \text{②}&: \\
5A - 4B &= 14 \qquad \text{④} \\
\text{①}+\text{④}&: \\
9A &= 26 \\
A &= \frac{26}{9} \\
\frac{26}{9} + B &= 3 \\
B &= \frac{1}{9} \\
-\frac{52}{9} + \frac{1}{9} + C &= -6 \\
C &= -\frac{1}{3} \\
\therefore \quad \frac{3x^2-6x+2}{(x-1)^2(x+2)} &= \frac{26}{9(x+2)} + \frac{1}{9(x-1)} - \frac{1}{3(x-1)^2}
\end{aligned}
$$

**r**

$$
\begin{aligned}
\frac{4}{(x-1)^2(2x+1)} &= \frac{A}{2x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \\
&= \frac{A(x-1)^2 + B(2x+1)(x-1) + C(2x+1)}{(x-1)^2(2x+1)} \\
&= \frac{Ax^2 - 2Ax + A + 2Bx^2 - Bx - B + 2Cx + C}{(x-1)^2(2x+1)}
\end{aligned}
$$

$$
\begin{aligned}
A + 2B &= 0 \qquad \text{①} \\
-2A - B + 2C &= 0 \qquad \text{②} \\
A - B + C &= 4 \\
2A - 2B + 2C &= 8 \qquad \text{③} \\
\text{③} - \text{②}&: \\
4A - B &= 8 \\
8A - 2B &= 16 \qquad \text{④} \\
\text{①} + \text{④}&: \\
9A &= 16 \\
A &= \frac{16}{9}
\end{aligned}
$$

$$\frac{16}{9} + 2B = 0$$

$$B = -\frac{8}{9}$$

$$\frac{16}{9} + \frac{8}{9} + C = 4$$

$$C = \frac{4}{3}$$

$$\therefore \quad \frac{4}{(x-1)^2(2x+1)} = \frac{16}{9(2x+1)} - \frac{8}{9(x-1)} + \frac{4}{3(x-1)^2}$$

**s** Divide:

$$\begin{array}{r} x - 2 \\ x^2 - 4 \overline{)\, x^3 - 2x^2 - 3x + 9} \\ \underline{x^3 - 0x^2 - 4x} \\ -2x^2 + x \\ \underline{-2x^2 + 8} \\ x + 1 \end{array}$$

$$\frac{x^3 - 2x^2 - 3x + 9}{x^2 - 4} = x - 2 + \frac{x+1}{x^2 - 4}$$

$$\frac{x+1}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2}$$

$$= \frac{A(x-2) + B(x+2)}{(x+2)(x-2)}$$

$$= \frac{Ax + Bx - 2A + 2B}{(x+2)(x-2)}$$

$$A + B = 1$$

$$2A + 2B = 2 \qquad (1)$$

$$-2A + 2B = 1 \qquad (2)$$

$(1) + (2)$:

$$4B = 3$$

$$B = \frac{3}{4}$$

$$A + \frac{3}{4} = 1$$

$$A = \frac{1}{4}$$

$$\therefore \quad \frac{x+1}{(x+2)(x-2)} = \frac{1}{4(x+2)} + \frac{3}{4(x-2)}$$

$$\frac{x^3 - 2x^2 - 3x + 9}{x^2 - 4} = x - 2 + \frac{1}{4(x+2)} + \frac{3}{4(x-2)}$$

**t** Divide:

$$\begin{array}{r} x \\ x^2 - 1 \overline{)\, x^3 + 3} \\ \underline{x^3 - x} \\ x + 3 \end{array}$$

$$\frac{x^3+3}{(x+1)(x-1)} = x + \frac{x+3}{(x+1)(x-1)}$$

$$\begin{aligned}\frac{x+3}{(x+1)(x-1)} &= \frac{A}{x+1} + \frac{B}{x-1} \\ &= \frac{A(x-1)+B(x+1)}{(x+1)(x-1)} \\ &= \frac{Ax+Bx-A+B}{(x+1)(x-1)}\end{aligned}$$

$$A + B = 1 \quad (1)$$

$$-A + B = 3 \quad (2)$$

$(1) + (2)$:

$$2B = 4$$

$$B = 2$$

$$A + 2 = 1$$

$$A = -1$$

$$\therefore \frac{x+3}{(x+1)(x-1)} = -\frac{1}{x+1} + \frac{2}{x-1}$$

$$\frac{x^3+3}{(x+1)(x-1)} = x - \frac{1}{x+1} + \frac{2}{x-1}$$

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$$\begin{aligned}\frac{2x-1}{(x+1)(3x+2)} &= \frac{A}{x+1} + \frac{B}{3x+2} \\ &= \frac{A(3x+2)+B(x+1)}{(x+1)(3x+2)} \\ &= \frac{3Ax+Bx+2A+B}{(x+1)(3x+2)}\end{aligned}$$

$$3A + B = 2 \quad (1)$$

$$2A + B = -1 \quad (2)$$

$(1) - (2)$: $A = 3$

$$9 + B = 2$$

$$B = -7$$

$$\therefore \frac{2x-1}{(x+1)(3x+2)} = \frac{3}{x+1} - \frac{7}{3x+2}$$